FRQ 4 Practice Focused on Advanced Trig

Practice FRQ 4 Problem Alligator

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 2^{x-5}$$
$$h(x) = \arctan(x+1)$$

i. Solve $g(x) = 12$ for all values of x in the domain of g.

ii. Solve $h(x) = -\frac{\pi}{6}$ for all values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \ln(2x^3) - \ln(10x^4) - 6\ln x$$
$$k(x) = \frac{1 - \sin^2 x}{\sin\left(x + \frac{3\pi}{2}\right)}$$

i. Rewrite $j(x)$ as a single natural logarithm without negative exponents in any part of the expression. Your result should be of the form $\ln(\text{expression})$.

ii. Rewrite $k(x)$ as a single term involving $\sec(x)$ and no other trigonometric function.

(C) The function m is given by $m(x) = \cos^2(3x) + 2\cos(3x)$. Find all input values in the domain of m that yield an output value of 3.

Practice FRQ 4 Problem Baboon

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 2\log_4(x) + 7$$
$$h(x) = 6\sin^2 x$$

i. Solve $g(x) = 8$ for all values of x in the domain of g.

ii. Solve $h(x) = 3$ for all values of x in the interval $[0, 2\pi)$.

(B) The functions j and k are given by

$$j(x) = \frac{4 \cdot 8^x}{2^{4x-1}}$$
$$k(x) = (\sec^2 x)\left(\frac{1}{2}\sin(2x) + \cos^2 x\right)$$

i. Rewrite $j(x)$ in the form $a(b)^x$.

ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

(C) The function m is given by $m(x) = \arccos(\tan x)$. Find all input values in the domain of m that yield an output value of $\frac{\pi}{2}$.

Practice FRQ 4 Problem Crocodile

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \log_4(3x)$$
$$h(x) = 4\sin^{-1}(x+1)$$

i. Solve $g(x) = -1$ for values of x in the domain of g.

ii. Solve $h(x) = \cos^{-1}\left(-\frac{1}{2}\right)$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = -\log_2(x-2) + 4\log_2(x+1) - \log_2(5)$$
$$k(x) = \frac{\cot^2 x}{\sin^2 x + \cot^2 x + \cos^2 x}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as a power of one trigonometric function.

(C) The function m is given by $m(x) = \sin(2x) - \sin(x)$. Find all input values in the domain of m that yield an output value of 0.

Practice FRQ 4 Problem Dolphin

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 6\arcsin x$$
$$h(x) = 2\log_4 x - \log_4 3$$

i. Solve $g(x) = -2\pi$ for values of x in the domain of g.
ii. Solve $h(x) = \log_4(12)$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = 3\log_2(x+5) - 1$$
$$k(x) = \frac{\cot x}{1 + \cot^2 x}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.
ii. Rewrite $k(x)$ as a multiple of $\sin(2x)$. Your answer should be of the form $a\sin(2x)$.

(C) The function m is given by $m(\theta) = \cos(2\theta) + 9\sin\theta$. Find all input values in the domain of m that yield an output value of -4.

Practice FRQ 4 Problem Elephant

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \ln(3x - 5)$$
$$h(x) = 2\tan^{-1}\left(\frac{x}{2}\right)$$

i. Solve $g(x) = 2$ for values of x in the domain of g.

ii. Solve $h(x) = \arccos\left(\frac{1}{2}\right)$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \csc\left(x + \frac{\pi}{2}\right)$$
$$k(x) = \frac{\sin(x - \pi)}{\left(\cos\left(x - \frac{\pi}{2}\right)\right)^2}$$

i. Rewrite $j(x)$ as an expression involving $\cos x$ and no other trigonometric functions.

ii. Rewrite $k(x)$ as an expression involving $\csc x$ and no other trigonometric functions.

(C) The function m is given by $m(x) = \sqrt{3}\cot\left(x - \frac{\pi}{2}\right)$. Find all input values in the domain of m that yield an output value of 3.

Practice FRQ 4 Problem Frog

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 5(4)^x$$
$$h(x) = \cos\left(x + \frac{\pi}{2}\right)$$

i. Solve $g(x) = 2$ for values of x in the domain of g.

ii. Solve $h(x) = -\frac{\sqrt{3}}{2}$ for values of x in the interval $\left[0, \frac{\pi}{2}\right)$.

(B) The functions j and k are given by

$$j(x) = (\sec^2 x)(\cos(2x))$$
$$k(x) = \frac{27 \cdot 9^x}{\sqrt{3^x}}$$

i. Rewrite $j(x)$ as an expression involving a power of $\tan x$ and no other trigonometric functions.

ii. Rewrite $k(x)$ as an expression of the form $3^{(ax+b)}$

(C) The function m is given by $m(x) = \sqrt{3}\cos(2x)$. Find all input values in the domain of m that yield an output value of $\frac{3}{2}$.

Practice FRQ 4 Problem Giraffe

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \tan(3x)$$
$$h(x) = 4e^{2x} + 5e$$

i. At what values of x does the graph of $y = g(x)$ have vertical asymptotes?

ii. Solve $h(x) = 6e$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \frac{1}{3}\log x - 2\log x$$
$$k(x) = 2\sin\left(x - \frac{\pi}{4}\right) - \sqrt{2}\sin x$$

i. Rewrite $j(x)$ as a single logarithm without negative exponents in any part of the expression. Your result should be of the form $\log(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\cos x$ appears once and no other trigonometric functions are involved.

(C) The function m is given by $m(x) = 4\csc(4x) - \sin(4x)$. Find all input values in the domain of m that yield an output value of 3.

Practice FRQ 4 Problem Hippopotamus

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \tan^2 x$$
$$h(x) = \log_5(2x - 1) - \log_5(x - 2)$$

i. Solve $g(x) = 3$ for values of x in the interval $\left[\frac{\pi}{2}, \pi\right)$.

ii. Solve $h(x) = 2$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \frac{(2^x \cdot 8^{x+1})^2}{4^x}$$
$$k(x) = \frac{4\cos^2 x - 2\cos(2x)}{\cot^2 x - \csc^2 x}$$

i. Rewrite $j(x)$ as an expression of the form 2^{ax+b}.

ii. $k(x)$ is equivalent to a constant value. Determine the value of this constant.

(C) The function m is given by $m(x) = 4\tan^{-1}(\sin(2x))$. Find all input values in the domain of m that yield an output value of π.

Answers to FRQ 4 Practice Focused on Advanced Trig

On the AP Exam, work must be shown where applicable.

Practice FRQ 4 Problem Alligator

(A)

i. $x = 5 + \log_2 12$

ii. $x = -\frac{\sqrt{3}}{3} - 1$

(B)

i. $\ln\left(\frac{1}{5x^7}\right)$

ii. $-\frac{1}{\sec x}$

(C) $x = \frac{2\pi k}{3}$, where k is any integer

Practice FRQ 4 Problem Baboon

(A)

i. $x = 2$

ii. $x = \frac{\pi}{4}, x = \frac{3\pi}{4}, x = \frac{5\pi}{4}, x = \frac{7\pi}{4}$

(B)

i. $8\left(\frac{1}{2}\right)^x$

ii. $\tan x + 1$

(C) $x = \pi k$, where k is any integer

Practice FRQ 4 Problem Crocodile

(A)

i. $x = \frac{1}{12}$

ii. $x = -\frac{1}{2}$

(B)

i. $\log_2\left(\frac{(x+1)^4}{5(x-2)}\right)$

ii. $\cos^2 x$

(C) $x = \pi k, x = \frac{\pi}{3} + 2\pi k, x = -\frac{\pi}{3} + 2\pi k$ where k is any integer

Practice FRQ 4 Problem Dolphin

(A)

i. $x = -\frac{\sqrt{3}}{2}$

ii. $x = 6$

(B)

i. $\log_2\left(\frac{(x+5)^3}{2}\right)$

ii. $\frac{1}{2}\sin(2x)$

(C) $\theta = \frac{7\pi}{6} + 2\pi k$, $\theta = \frac{11\pi}{6} + 2\pi k$ where $k \in \mathbb{Z}$

Practice FRQ 4 Problem Elephant

(A)

i. $x = \frac{e^2+5}{3}$

ii. $x = \frac{2\sqrt{3}}{3}$

(B)

i. $\frac{1}{\cos x}$

ii. $-\csc x$

(C) $x = \frac{2\pi}{3} + \pi k$, where k is any integer

Practice FRQ 4 Problem Frog

(A)

i. $x = \log_4\left(\frac{2}{5}\right)$

ii. $x = \frac{\pi}{3}$

(B)

i. $1 - \tan^2 x$

ii. $3^{\left(\frac{3}{2}x+3\right)}$

(C) $x = \frac{\pi}{12} + \pi k$, $x = \frac{11\pi}{12} + \pi k$, where k is any integer

Practice FRQ 4 Problem Giraffe

(A)

i. $x = \frac{\pi}{6} + \frac{\pi}{3}k$, for $k \in \mathbb{Z}$

ii. $x = \frac{1}{2}\ln\left(\frac{e}{4}\right)$

(B)

i. $\log\left(\frac{1}{x^{\frac{5}{3}}}\right)$

ii. $-\sqrt{2}\cos x$

(C) $x = \frac{\pi}{8} + \frac{\pi k}{2}$, for any integer k

Practice FRQ 4 Problem Hippopotamus

(A)

i. $x = \frac{2\pi}{3}$

ii. $x = \frac{49}{23}$

(B)

i. 2^{6x+6}

ii. -2

(C) $x = \frac{\pi}{4} + \pi k$, $k \in \mathbb{Z}$